



AUTORES AUTHORS	PALAVRAS CHAVES/KEY WORDS		AUTORIZADA POR/AUTHORIZED BY
	MODELO DIGITAL DE ELEVAÇÃO INTERPOLAÇÃO NUMÉRICA	SPLINES BETA-SPLINES	<i>Múcio Roberto Dias</i> Múcio Roberto Dias Vice Director General

AUTOR RESPONSÁVEL RESPONSIBLE AUTHOR	DISTRIBUIÇÃO/DISTRIBUTION	REVISADA POR / REVISED BY
<i>Carlos Eduardo Nery</i> Carlos Eduardo Nery	<input type="checkbox"/> INTERNA / INTERNAL <input checked="" type="checkbox"/> EXTERNA / EXTERNAL <input type="checkbox"/> RESTRITA / RESTRICTED	

CDU/UDC	DATA / DATE
519.65	Março 1990

TÍTULO/TITLE	PUBLICAÇÃO Nº PUBLICACION NO		ORIGEM ORIGIN
	BETA SPLINES INTERPOLATION FOR DEM'S		LAC
AUTORES/AUTHORSHIP	Carlos Eduardo Nery Luiz Alberto Vieira Dias		PROJETO PROJECT
			INBM
		Nº DE PAG. NO OF PAGES	ULTIMA PAG. LAST PAGE
		13	10
		VERSÃO VERSION	Nº DE MAPAS NO OF MAPS

RESUMO - NOTAS / ABSTRACT - NOTES

The interpolators used for the densification of Digital Elevation Models normally do not take into account additional information known to the operator, like rivers, lakes (flat areas), or mountain ridges. They interpolate based on the control points only. The Beta-splines curves, on the other hand, have properties that enable the operator to modify the bias and the tension for each cell by assigning values to the parameters beta1 and beta2. Thus, it is possible to force the resulting three dimensional surface to acquire a desirable shape. The disadvantage is the relatively large amount of extra calculations, but with the now available faster, and cheaper microcomputers, this disadvantage can be overcome. This work tests the use of Beta-splines interpolators for DEM's running on an IBM-PC-like environment.

OBSERVAÇÕES / REMARKS

This paper will be submitted to the "International Society of Photogrammetry and Remote Sensing (ISPRS)", Tsukuba, Japan, May 15-18, 1990.

## RESUMO

Os interpoladores usados para a densificação de Modelos Digitais de Elevação normalmente não levam em conta informações adicionais do conhecimento do operador, como rios, lagos (áreas planas), ou cristas de montanhas. Eles interpolam baseado apenas nos pontos de controle. As curvas Beta-splines, por outro lado, têm propriedades que permitem ao operador modificar o viés e a tensão para cada cédula, por meio de uma atribuição de valores aos parâmetros  $\beta_1$  e  $\beta_2$ . Então, é possível forçar a superfície tridimensional resultante a adquirir uma forma desejada. A desvantagem é o volume relativamente grande de cálculos extra necessários, mas com os mais rápidos e baratos microcomputadores já disponíveis esta desvantagem desaparece. Este trabalho testa o uso de interpoladores Beta-splines para Modelos Digitais de Elevação rodando em máquinas do tipo IBM-PC-XT.



# BETA-SPLINES INTERPOLATION FOR DIGITAL ELEVATION MODELS

Carlos Eduardo Nery  
Luiz Alberto Vieira Dias

Institute for Space Research - INPE  
Av. dos Astronautas, 1758 - 12225 S.J.Campos SP BRAZIL

## Abstract

The interpolators used for the densification of Digital Elevation Models normally do not take into account additional information known to the operator, like rivers, lakes (flat areas), or mountain ridges. They interpolate based on the control points only. The Beta-spline curves, on the other hand, have properties that enable the operator to modify the bias and the tension for each cell by assigning values to the parameters  $\beta_1$  and  $\beta_2$ . Thus, it is possible to force the resulting three dimensional surface to acquire a desirable shape. The disadvantage is the relatively large amount of extra calculations, but with the now available faster, and cheaper microcomputers, this disadvantage can be overcome. This work tests the use of Beta-splines interpolators for DEM's running on an IBM-PC-like environment.

## 1. INTRODUCTION

The Institute for Space Research - INPE - developed a Geographical Information System (GIS) based on an IBM-PC-like environment (1). One of the possible operations this system performs is Digital Elevation Models (DEM) manipulations, like the change in scale and visualization from different observer positions. In order to achieve acceptable results, it is necessary to use a good interpolator. The GIS of INPE already has two interpolators available: linear, and Akima. The first one is not proper for certain applications, due to the lack of smoothness, while the other is excellent for certain cases. However when additional information about the terrain is known, like flat areas or sharp edges, the above interpolators do not perform properly. An interpolator, the Beta-spline, allows the operator to reshape the resulting surface by allotting different values for parameters  $\beta_1$  and  $\beta_2$ , that change the surface bias and tension, respectively, in each cell.

The Beta-spline program was developed, on C computer language, to be added to the INPE's GIS. The main objective of this work is to test a first version of this software, for the evaluation of its performance, especially with respect to the use of a microcomputer, for DEM applications.

Next section presents a brief outline of the Beta-spline interpolator, the following the results obtained with the use of this technique applied to examples, finally some conclusions are drawn, regarding microcomputer performance.

## 2. BETA-SPLINES

Beta-splines are generalizations of the known B-splines (2). They can be represented parametrically by:

$$Q_{ij}(\bar{u}, \bar{v}) = \sum_{i,j} V_{ij} B_{ij}(\bar{u}, \bar{v})$$

where the  $V$ 's are the control vertices, and  $B_{ij}(\bar{u}, \bar{v})$  are basis functions in two independent parameters,  $\bar{u}$  and  $\bar{v}$ , that are nonzero locally, that are non-negative, and that sum to one.

Bartels et al. divide the basis function into two independent basis functions, each depending on just one of the space parameters  $u$  or  $v$ . Thus, the Beta-spline formula becomes, with  $u = \bar{u} - u_i$ , and  $v = \bar{v} - v_i$ , according to Bartels et al. (2).

$$Q_{ij}(u, v) = \sum_r \sum_s V_{i+r, j+s} b_r(u) b_s(v)$$

with  $r$  and  $s$  varying in the range  $-3, \dots, 0$ , for each patch (or cell).  $Q_{ij}(u, v)$ , the  $i$ th,  $j$ th patch is determined by 16 control vertices. The basis functions  $b_r(u)$ , and  $b_s(v)$  have the same form and are dependent on the parameters  $\beta_1$  and  $\beta_2$ . It is possible to use different  $\beta_1$  and  $\beta_2$  for each of the functions and for each cell. The generic formulas for the  $b$ 's are:

$$b(-0)(x) = 1/d * [2u^3]$$

$$b(-1)(x) = 1/d * [2 + (6\beta_1)u + (3\beta_2 + 6(\beta_1)^2)u^2 - (2\beta_2 + 2(\beta_1)^2 + 2\beta_1 + 2)u^3]$$

$$b(-2)(x) = 1/d * [(2\beta_2 + 4(\beta_1)^2 + 4\beta_1) + (6(\beta_1)^3 - 6\beta_1)u - (3\beta_2 + 6(\beta_1)^3 + 6(\beta_1)^2)u^2 + (2\beta_2 + 2(\beta_1)^3 + 2(\beta_1)^2 + 2\beta_1)u^3]$$

$$b(-3)(x) = 1/d * [2(\beta_1)^3 - (6(\beta_1)^3)u + 6(\beta_1)^3u^2 - 2(\beta_1)^3u^3]$$

and

$$d = \beta^2 + 2(\beta_1)^3 + 4(\beta_1)^2 + 4\beta_1 + 2 \neq 0$$

If  $\beta_1$  is made equal to 1 and  $\beta_2$  equal to zero, the B-spline formulas are recovered.

### 3. RESULTS

In order to test the Beta-spline's performance, it was developed on an IBM-PC-like ambient a software to generate the surface and display it.

It was chosen a DEM from a real terrain, near São José dos Campos SP, Brazil. Six examples with different  $\beta_1$  and  $\beta_2$  pairs were used to demonstrate the parameters effect on the DEM. It is possible to use different parameters for each patch and each direction (u and v). However, on this work, it was not attempted to do it. It were used the same betas for the whole DEM. however, since a wide range of possible betas combinations were tried, it is hoped that it was enough to assess the Beta-splines capabilities.

Figure 1 is an interpolation with  $\beta_1$  equal to one and  $\beta_2$  equal to zero. This corresponds to the B-spline interpolator (2). Table 1 presents a summary of all the tested cases.

TABLE 1

Case	$\beta_1$	$\beta_2$	Figure
A	1	0	1
B	8	0	2
C	1	25	3
D	1000	10	4
E	1	8	5
F	0.0001	10	6

The parameter  $\beta_1$  is responsible for the bias. One means a centered surface patch around the control vertices, while less than one means a distortion towards u and v near zero, and larger than one values a distortion towards u and v near 1. As far as  $\beta_2$ , zero means that the tension is the same as in a B-spline surface, while larger values indicates that the curve passes closer to the control vertices. When  $\beta_2$  is infinity, the interpolation becomes a linear interpolation, if  $\beta_1$  is one (2).

Cases A, C, and E are without bias, and A and B without tension. Figures 2 to 6 present the effect of the use of Beta-splines on the DEM under study.

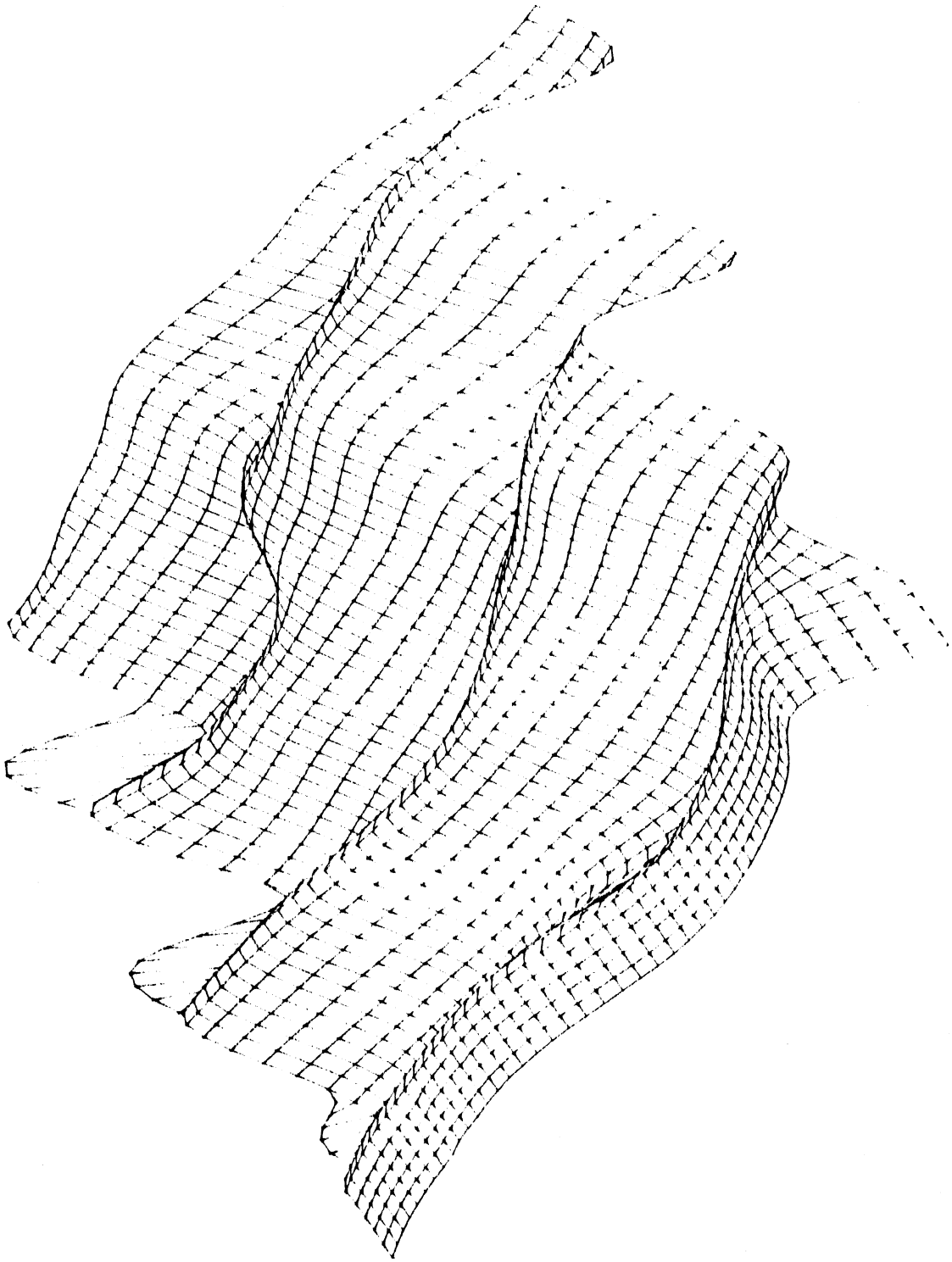


Fig. 1 -  $\beta_1 = 1$ ;  $\beta_2 = 0$



Fig. 2 -  $\beta_1 = 8$ ;  $\beta_2 = \emptyset$





Fig. 3 -  $\beta_1 = 1$ ;  $\beta_2 = 25$



Fig. 4 -  $\beta_1 = 1000$ ;  $\beta_2 = 10$

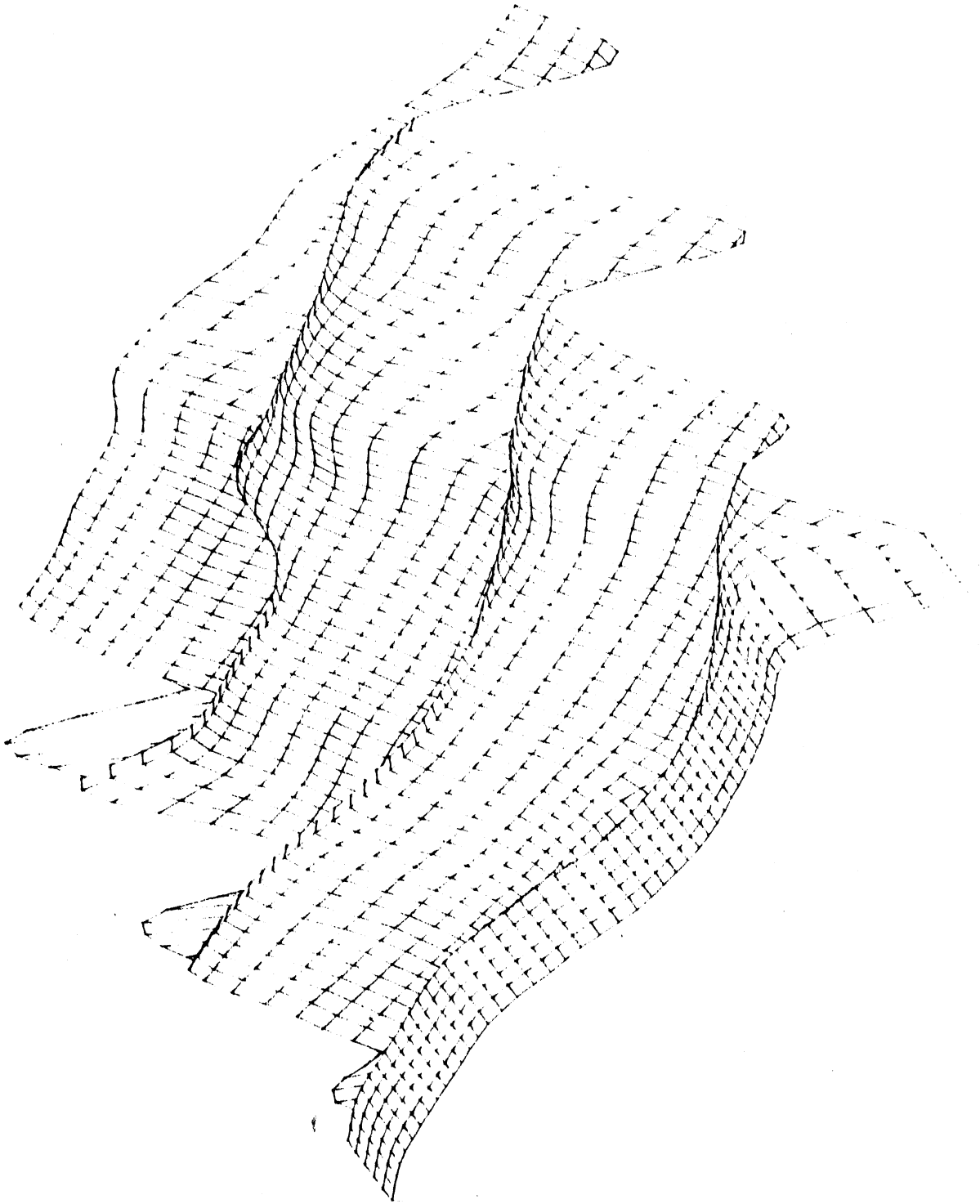


Fig. 5 -  $\beta_1 = 1$ ;  $\beta_2 = 8$



Fig. 6 -  $\beta_1 0.001$ ;  $\beta_2 = 10$

#### 4. CONCLUSIONS

One of the main objectives of this work was to test the microcomputer behavior, since long calculations have to be performed. The hardware used for the display was the SITIM, an INPE developed image processing microcomputer based system. The calculations were made on two types of microcomputers, on an IBM-PC-XT without arithmetic coprocessor, that took 36 minutes, and on an IBM-PC-AT with arithmetic coprocessor, in which the computations for each DEM were made in 5 minutes and 40 seconds. Since the code is not optimized these times can be reduced. The plots were made with the help of the Fundação Valeparaibana de Ensino SITIM System. The authors would like also to thanks INPE and IBM Rio Scientific Center for the support, since this work is part of a joint project been run by both institutions.

#### REFERENCES

1. Alves, D.S.; Camara-Neto, G.; Erthal, G.J.; Felgueiras, C.A.; Paiva, J.A.; Oliveira, E.A.; Dias, L.A.V.; Godoy, M.; Abraão, A. (1988), Sistema de Informações Geográficas (Geographic Information System)". Internal Report, INPE, São José dos Campos, SP, Brazil.
2. Bartels, R.H.; Beatty, J.C.; Barsky, B.A. (1986), "Introduction to the Use of Splines in Freeform Curve and Surface Design". Course E4, ACM SIGGRAPH 1986 publication, Dallas, Texas.